PHYSICS

1. $v_y = \sqrt{\frac{T_y}{\mu_y}}$

$T_y = \left\{\int_0^y \mu_0 e^y dy\right\} g$

$T_y = \mu_0(e^y - 1).g$

$v_y = \sqrt{g - \frac{g}{e^y}}$

$v_y^2 = g(1 - e^{-y})$.

2. Path difference = Path B – Path A
= 7d – 3d = 4d
[Note that there is no phase change in reflections from mirror in case of sound]
For being out of phase :

$\Delta x = 4d = \frac{\lambda}{2}\ ;\ \frac{3\lambda}{2}$;................

For minimum d, $4d = \frac{\lambda}{2}$

$\Rightarrow$ $d = \frac{\lambda}{8}$ **Ans.**

3. For interference at A : S_2 is behind of S_1 by a distance of $100\lambda + \frac{\lambda}{4}$.(equal to phase difference $\frac{\pi}{2}$). Further S_2 lags S_1 by $\frac{\pi}{2}$. Hence the waves from S_1 and S_2 interfere at A with a phase difference of $200.5\,\pi + 0.5\pi = 201\pi = \pi$

Hence the net amplitude at A is 2a – a = a

For interference at B : S_2 is ahead of S_1 by a distance of $100\lambda + \frac{\lambda}{4}$.(equal to phase difference $\frac{\pi}{2}$). Further S_2 lags S_1 by $\frac{\pi}{2}$.

Hence waves from S_1 and S_2 interfere at B with a phase difference of $200.5\,\pi - 0.5\pi = 200\pi = 0\pi$.
Hence the net amplitude at A is 2a + a = 3a

Hence $\left(\frac{I_A}{I_B}\right) = \left(\frac{a}{3a}\right)^2 = \frac{1}{9}$

4. $\dfrac{V}{4(\ell_1 + e)} = f, \quad \dfrac{3V}{4(\ell_2 + e)} = f$

$\dfrac{V}{4f} = \ell_1 + e$

$\dfrac{3V}{4f} = \ell_2 + e$

$\dfrac{2V}{4f} = \ell_2 - \ell_1 = V = 2f\,(\ell_2 - \ell_1)$

$e = \dfrac{2f(\ell_2 - \ell_1)}{4f} - \ell_1 = \dfrac{2\ell_2 - 2\ell_1 - 4\ell_1}{4} = \dfrac{2(\ell_2 - 3\ell_1)}{4}$

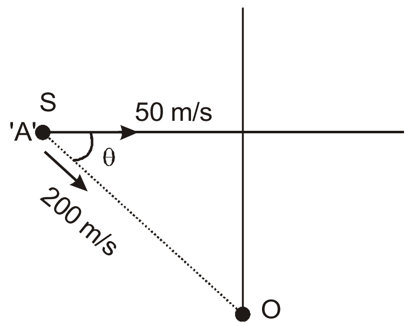
5. $f = f_0\left(1 + \dfrac{V_{ob}}{V_{sound}}\right)$

$\Rightarrow \dfrac{f}{f_0} = 1 + \dfrac{V_{ob}}{V_{sound}}$ (straight line) ; when $\dfrac{V_{ob}}{V_{sound}} = 0$; $\dfrac{f}{f_0} = 1$.

and as $\dfrac{V_{ob}}{V_{sound}} \to 1 \quad \Rightarrow \quad \dfrac{f}{f_0} \to 2$

6. Sound emitted by source at S which is observed by observer when source crosses origin.

Then $\cos\theta = \dfrac{50t}{200t} = \dfrac{1}{4}$



$96 = f\left(\dfrac{200 - 0}{200 - 50\cos\theta}\right)$

f = 90 Hz

7. When we move along +x direction (top to bottom) x increases but T decreases

$v_{wave} = \sqrt{\dfrac{T}{\mu}}$

when T decreases μ must decrease

$\Rightarrow \quad x \to$ increases $\quad \mu \to$ decreases $\Rightarrow \quad \dfrac{d\mu}{dx} < 0$

8. $f = \dfrac{v}{2(\ell + 2e)}$ where e = end correction = 0.6 r

$\therefore \quad f = \dfrac{v}{2(\ell + 2 \times 0.6r)} = \dfrac{v}{2(\ell + 1.2r)}$

$\therefore \quad \dfrac{\Delta f}{f} = \dfrac{\Delta v}{v} - \dfrac{\Delta(\ell + 1.2r)}{\ell + 1.2r} = \dfrac{\Delta v}{v} - \dfrac{\Delta\ell + 1.2\Delta r}{\ell + 1.2r}$

here $\dfrac{\Delta v}{v}$ = 0 (given) $\dfrac{\Delta f}{f} \times 100 = -\dfrac{\Delta\ell + 1.2\Delta r}{\ell + 1.2r} \times 100$

for maximum % error : $\Delta\ell = 0.1$, $\Delta r = 0.05$

$\left(\dfrac{\Delta f}{f} \times 100\right)_{max} = \dfrac{0.1 + 1.2 \times 0.05}{94 + 1.2 \times 5} \times 100$ = **0.16% ...Ans.**

9. $f = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{\mu}}$

If radius is doubled and length is doubled, mass per unit length will become four times. Hence

$f' = \dfrac{1}{2 \times 2\ell}\sqrt{\dfrac{2T}{4\mu}} = \dfrac{f}{2\sqrt{2}}$

10. $L = \dfrac{m\lambda_1}{2}$ and $L\ (m + 1)\ \dfrac{\lambda_2}{2}$

Where m is no. of harmonic

m.36 = (m + 1) 32 $\Rightarrow$ m = 8

L = 8 × 18 = 144 cm

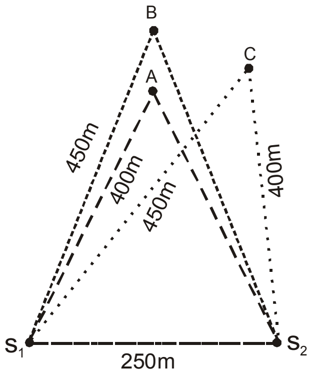
11. (C) $P = \dfrac{1}{2}\mu\omega^2 A^2 V$

using $V = \sqrt{\dfrac{T}{\mu}}$

$P = \dfrac{1}{2}\omega^2 A^2 \sqrt{T\mu}$

$\omega = \sqrt{\dfrac{2P}{A^2\sqrt{T\mu}}} \quad f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{2P}{A^2\sqrt{T\mu}}}$

using data f = 30 Hz.

12. 

At points A and B, path difference between the waves coming from two radio stations is zero. Hence there will be constructive interference at A and B,

For point C, path difference between the waves is 50 metre i.e. $\frac{\lambda}{2}$ so destructive interference takes places at point C.

13. $P_0 = BKS_0$; $k = \frac{2\pi}{\lambda}$; $\lambda = \frac{v}{f}$; $v = \sqrt{\frac{B}{\rho}}$

Using above, we get

$$S_0 = \frac{P_0}{2\rho v\pi f} = \frac{5}{2\times1\times330\times3.14\times1875}$$

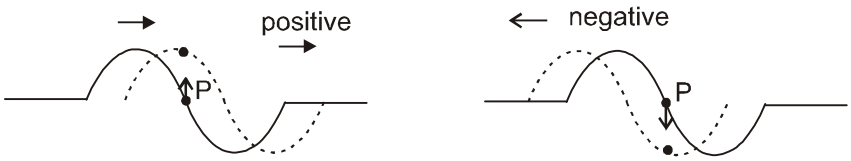
$\simeq$ 1 μ meter.

14. Wavelength remains same during approach and recede.

15. The period of beats is the time between maximum intensities. The square of the pressure is proportional to the intensity.

Beat frequency = $\frac{1}{\text{Beat period}} = \frac{1}{0.1}$ = 10 Hz.

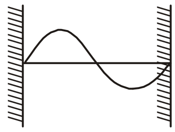
16. $V = \sqrt{\frac{T}{\lambda}} = \frac{\omega}{k} \Rightarrow T = \frac{\omega^2\lambda}{k^2} = \left(\frac{420}{21}\right)^2 \times 0.2 = 80$ N.

17. 

18. Speed of wave in wire $V = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell}A \times \frac{1}{\rho A}} = \sqrt{\frac{Y\Delta\ell}{\ell\rho}}$

Maximum time period means minimum frequency ; that means fundamental mode.

$$f = \frac{V}{\lambda} = \frac{V}{2\ell}$$



$\ell = \lambda$

$\therefore \quad T = \frac{2\ell}{V} = 2\ell \sqrt{\frac{\ell\rho}{Y\Delta\ell}} = \frac{1}{35}$ second **Ans.**

$\therefore$ (f = 35 Hz)

and; frequency of first overtone = $\frac{V}{\ell}$ = 70 Hz.

19. $\frac{\lambda}{4} = 0.1 \qquad \Rightarrow \lambda = 0.4$ m

from graph $\Rightarrow$ T = 0.2 sec. and amplitude of standing wave is 2A = 4 cm.

Equation of the standing wave

$$y(x,\ t) = -2A \cos\left(\frac{2\pi}{0.4}x\right).\sin\left(\frac{2\pi}{0.2}t\right) \text{ cm}$$

y (x = 0.05, t = 0.05) = $-2\sqrt{2}$ cm

y(x = 0.04, t = 0.025) = $-2\sqrt{2}$ cos 36°

speed = $\frac{\lambda}{T}$ = 2 m/sec.

$$V_y = \frac{dy}{dt} = -2A \times \frac{2\pi}{0.2} \cos\left(\frac{2\pi x}{0.4}\right).\cos\left(\frac{2\pi t}{0.2}\right)$$

V_y = (x = $\frac{1}{15}$ m, t = 0.1) = 20 π cm/sec.

20. Frequency of horn directly heard by observer $\frac{v + v_0}{v + v_c}f$

Frequency of echo = $\frac{v}{v + v_c}f$

Frequency of echo of horn as heard by observer.

$$\frac{v}{v - v_c}f.\left(\frac{v + v_0}{v}\right)$$

Frquency of Beats :

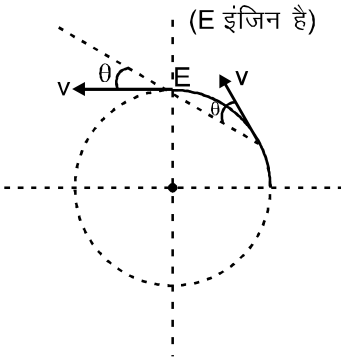
$$= (v + v_0)\, f \left\{\frac{1}{v - v_c} - \frac{1}{v + v_c}\right\} = \frac{2v_c(v + v_0)}{(v^2 - v_c^2)}f$$

21. $f = 5.\frac{v}{4\ell}$

$\Rightarrow \quad \ell = \frac{5v}{4f} = \frac{15}{16}$ m

The open end is position of node of pressure. There is no pressure variation.

22.



$$f_{obs} = \frac{f\,[v_s + v\cos\theta]}{[v_s + v\cos\theta]} = f$$

$$\lambda_{obs} = \frac{v_s + v\cos\theta}{f}$$

For any observer in train frequency observed is equal to original frequency but observed wavelength is more.

23. $C = 325\ \frac{m}{sec}$

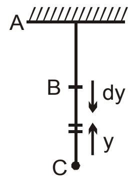
f = 600 Hz

$$f_A = \left(\frac{C-V_A}{C+V_S}\right) f = \frac{3600}{7}\ \text{Hz}$$

$$f_B = \left(\frac{C+V_B}{C+V_S}\right) f = 600\ \text{Hz}$$

$$f_C = \left(\frac{C+V_c}{C-V_S}\right) f = 700\ \text{Hz}$$

24. For part BC



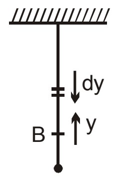
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{m/\ell . y.g}{m/\ell}} = \sqrt{y.g}$$

$$\Rightarrow \quad \frac{dy}{dt} = \sqrt{y.g} \quad \Rightarrow \quad \int_0^{\ell} \frac{dy}{\sqrt{y}} = \int_0^{t_1} \sqrt{g}\, dt$$

$$\Rightarrow \quad 2\sqrt{\ell} = \sqrt{g}.t_1$$

$\Rightarrow$ t_1 = time to go from C to B $= 2\sqrt{\frac{\ell}{g}}$

For part BA



$$v = \sqrt{\frac{2mg + \frac{m}{\ell}.y.g}{m/\ell}} = \sqrt{(2\ell + y)g}$$

$$\frac{dy}{dt} = \sqrt{(2\ell + y)g} \quad \Rightarrow \quad \int_0^{\ell} \frac{dy}{\sqrt{2\ell + y}} = \int_0^{t_2} \sqrt{g}\, dt$$

$$\Rightarrow \quad 2(\sqrt{3} - \sqrt{2}).\sqrt{\ell} = \sqrt{g}.t_2$$

$\Rightarrow$ t_2 = time to go from B to A = $2(\sqrt{3} - \sqrt{2}).\sqrt{\frac{\ell}{g}}$

$\therefore$ total time = $t_1 + t_2 = 2\sqrt{\frac{\ell}{g}} + 2(\sqrt{3} - \sqrt{2}).\sqrt{\frac{\ell}{g}}$

25. Total energy $E = \int_0^{\ell} \frac{1}{2} dmv^2$

$$= \int_0^{\ell} \frac{1}{2} dm A_x{}^2 \omega^2 = \int_0^{\ell} \frac{1}{2}\left(\frac{m}{\ell}\right) dx.A^2 \sin^2 kx.\omega^2 = \frac{1}{4} mA^2\omega^2$$

$$\omega = 2\pi f = 2\pi.\frac{3v}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{3\pi}{\ell}\sqrt{\frac{T\ell}{m}}$$

$\therefore$ Energy $= \frac{1}{4} ma^2.\frac{9\pi^2}{\ell^2}.\frac{T\ell}{m}$

Energy $= \frac{9}{4}\frac{a^2\pi^2 T}{\ell}$

So, energy between two consecutive nodes = $\frac{3}{4}\frac{a^2\pi^2 T}{\ell}$.

26. $y = 0.10 \sin\left(\frac{\pi x}{3}\right) \sin(12\pi t)$

$k = \frac{\pi}{3} \quad \Rightarrow \quad \lambda = 6m$

Length of the rope = λ = 6m

$$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t) = 2 \sin\left(\frac{\pi x}{15} + 96\pi t\right) + 2 \sin\left(\frac{\pi x}{15} - 96\pi t\right)$$

27. Sound level in dB is

$B = 10 \log_{10}\left(\frac{I}{I_0}\right)$

If B_1 and B_2 are the sound levels and I_1 and I_2 are the intenaities in the two cases

$B_2 - B_1 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$

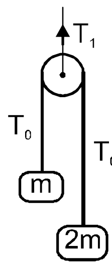
$\frac{I_2}{I_1} = 100 \qquad$ So $\qquad \frac{S_{02}}{S_{01}} = \sqrt{\frac{I_2}{I_1}} = 10$

$S_{02} = 10\, S_{o1}$

and $\quad S_{01} = \frac{P_0}{BK} = \frac{3}{2} \times \frac{10^{-3}}{1.5 \times 10^5} \times \frac{20\pi \times 10^{-2}}{2\pi} = 10Å$

So $\quad S_{o2} = 100Å$

28. $T_1 = 2T_0 = 2\left[\frac{2m(2m)}{m + 2m}\right] g$



$T_1 = \frac{8m}{3} g = \frac{80m}{3}$ ....(i)

In resonance,

$f_{wire} = f_{tube}$

$\frac{(1)V_1}{2\ell_1} = \frac{(1)V_2}{4\ell_2}$

$$\frac{\left(\sqrt{\frac{T_1}{\mu}}\right)}{2(x)} = \frac{(400)}{4\left(\frac{x}{2}\right)}$$

$\Rightarrow \quad T_1 = \mu(16 \times 10^4)$

From (i), $\quad \frac{80}{3} m = 10^{-4} (16 \times 10^4)$

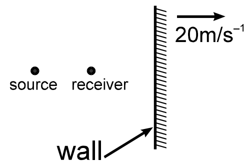
$m = 0.6$ kg.

29. As ; $f_1 = f$ (For direct sound)

Now ; for reflected sound $f_2 = \left(\frac{V-20}{V+20}\right)f$

If b is the beat frequency ;

$\therefore b = f_1 - f_2$



$\therefore\ f - \left(\frac{V-20}{V+20}\right)f \quad = \frac{f.40}{v+20}$

$= \frac{300.40}{350} = \frac{240}{7}$ Hz

30. $v = \sqrt{\frac{\gamma RT}{M}}$

$M = \frac{4\times 2 + 2\times 2}{4} = 3g$

$\gamma = 1 + \frac{2}{f} = 1 + \frac{2\times(2+2)}{2\times 3 + 2\times 5} = \frac{3}{2}$

$\therefore \quad v = \sqrt{\frac{3}{2}\times\frac{25}{3}\times\frac{1000}{3}\times\frac{972}{5}} = 900$ m/s

Ans. 90

31. Imagine a cylinder of radius 7m and length 10m. Intensity of sound at the surface of cylinder is same everywhere.

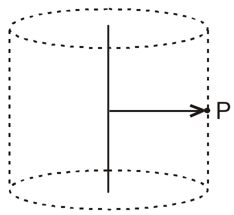
Therefore $\quad I = \frac{P}{2\pi rL} \quad = \frac{2.2\times 10^4}{2\pi\times 10\times 7}$

(As sound is propagating radially out only, sound energy does not flow out through the ends)

$\therefore \quad I = 50\ W/m^2$

Energy intercepted by the detector

$= I \times A \quad = 12$ mW



32. $\lambda = 2\ell = 3m$

Equation of standing wave

$y = 2A \sin kx \cos \omega t$

$y = A$ as amplitude is 2A.

$A = 2A \sin kx$

$\frac{2\pi}{\lambda}x = \frac{\pi}{6} \quad \Rightarrow \quad x_1 = \frac{1}{4}m$

and $\frac{2\pi}{\lambda}.x = \frac{5\pi}{6} \quad \Rightarrow \quad x_2 = 1.25\ m \Rightarrow x_2 - x_1 = 1m$

33 to 34

(33) $f_{1i} = f_{1r} = \frac{v}{v - v_c} f$, $f_{2i} = f_{2r} = \frac{v}{v + v_c} f$

Now, for driver $f_{dr1} = \frac{v + v_c}{v} f_{1r}$

and $f_{dr2} = \frac{v - v_c}{v} f_{2r}$

So, beat frequency= $| f_{dr1} - f_{dr2} |$

$$= \left| \frac{v + v_c}{v} f_{1r} - \frac{v - v_c}{v} f_{2r} \right| = \left\{ \frac{(v + v_c)^2 - (v - v_c)^2}{(v + v_c)(v - v_c)} \right\} f$$

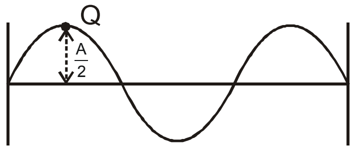
$$= \left(\frac{4vv_c}{v^2} \right) f = \left(\frac{4v_c}{v} \right) f .$$

(34) $\lambda_1 = \frac{v + v_c}{f}$ $\lambda_2 = \frac{v - v_c}{f}$

$\lambda_1 - \lambda_2 = \frac{2v_c}{f}$, $\lambda_1 + \lambda_2 = \frac{2v}{f}$

$$\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \frac{v_c}{v} .$$

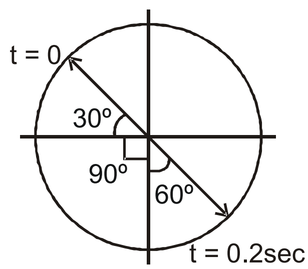
35 to 37 t = 0



Displacement equations of point Q = $A \sin \left(\omega t + \frac{5\pi}{6} \right)$

Equation of standing wave $y(x) = A(x) \sin \left(\omega t + \frac{5\pi}{6} \right) = A \sin kx . \sin (\omega t + \frac{5\pi}{6})$

According to snapshots



$t = \frac{1}{5} = \frac{\pi}{\omega} \Rightarrow \omega = 5\pi$ rad/s

Time period $T = \frac{2\pi}{5\pi} = \frac{2}{5}$ sec

wavelength $\lambda = 0.2$ m

wave velocity $v = \frac{\lambda}{T} = \frac{2}{10} \cdot \frac{5}{2} = \frac{1}{2}$ m / s

Disp. equation for point P $y = A \sin \left(\omega t + \frac{11\pi}{6} \right)$

velocity equation for point P $V_p = \omega A \cos \left(\omega t + \frac{11\pi}{6} \right)$

Acceleration equation for point P $a_p = -\omega^2 A \sin \left(\omega t + \frac{11\pi}{6} \right)$

here $\omega = 5\pi$rad/s A = 2 cm

38. $f = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$

$f \propto \frac{\sqrt{T}}{\ell}$

39. $f \propto \sqrt{T}$

So Δf increases by increasing T.

i.e. $f_2 = f_1 + 3 = 443$ Hz]

40 to 42

Applying cosine rule in the triangle S_1S_2A, $\cos 60^0 = \frac{3^2 + 4^2 - S_1A^2}{2 \times 3 \times 4} \Rightarrow S_1A = \sqrt{13}$ .For line sources intensity is inversely proportional to the distance from the source. At A , let the intensity due to the source S_1 be I, then $I\sqrt{13} = I_0 3$.

Similarly at B , let the intensity due to the source S_1 be I', then $I'5 = I_0$ 3. Path difference = 2m = 2 λ.

$\therefore$ the interference will be constructive. $\therefore I_{res} = I_0 + I' + 2\sqrt{I_0 I'}$

43. (A) The fundamental frequency in the string,

$$f_0 = \frac{\sqrt{T/\mu}}{2\ell} = \sqrt{\frac{102.4}{1 \times 10^{-3}}} \times \frac{1}{2 \times 0.5} \text{ Hz } = 320 \text{ Hz.}$$

Other possible resonance frequencies are f_A and f_0 = 320 Hz, 640 Hz, 960 Hz.

(B) The fundamental frequency in the string.

$$f_0 = \frac{\sqrt{T/\mu}}{4\ell} = \frac{320}{4 \times 0.5} = 160 \text{ Hz.}$$

Other possible resonance frequencies are

f_B = 160 Hz, 480 Hz, 800 Hz.

(C) The fundamental frequency in both ends open organ pipe is

$$f_0 = \frac{v}{2\ell} = \frac{320}{2 \times 0.5} = 320 \text{ Hz.}$$

Other possible resonance frequencies are

f_c = 320 Hz, 640 Hz, 960 Hz

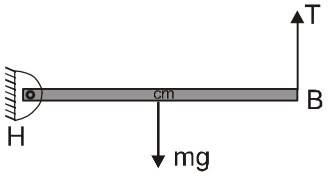
(D) The fundamental frequency in one end open organ pipe is

$$f_0 = \frac{v}{4\ell} = \frac{320}{4 \times 0.5} = 160 \text{ Hz.}$$

Other possible resonance frequencies are

f_D = 160 Hz, 480 Hz, 800 Hz.

45. (A) Due to reflection from a moving wall, frequency of the sound wave will change. So, the superposition of the incident waves and the reflected waves will produce beats.

(B) 

Applying torque balance about the hinge point 'H'

$$(mg)\left(\frac{\ell}{2}\right) = (T)(\ell)$$

$$T = \frac{mg}{2} = \frac{20 \times 10}{2} = 100 \text{ m}$$

Natural frequencies of the fixed–free wire are

$$f = \frac{1}{4\ell}\sqrt{\frac{T}{\mu}},\ \frac{3}{4\ell}\sqrt{\frac{T}{\mu}},\ \frac{5}{4\ell}\sqrt{\frac{T}{\mu}},\ldots\ldots\ldots\ldots\ldots$$

$$f = \frac{1}{4 \times 1}\sqrt{\frac{100}{0.01}},\ \ldots\ldots\ldots\ldots\ldots \quad \Rightarrow \quad f = 25,\ \underline{\boldsymbol{75}},\ 125,\ldots\ldots\ldots\ldots\ldots$$

f = 75 Hz matches with the frequency of the source, so resonance will occur and standing waves are generated.

(C) $y = A\sin^2(\omega t - kx) + B\cos^2(kx - \omega t) + C\cos(kx + \omega t)\sin(kx + \omega t)$

Solving we can get,

y = (some constant) $\cos 2(\omega t - kx)$ + (some constant) $\sin 2(kx + \omega t)$

which is superposition of waves moving in opposite direction. So, standing waves can be produced.

But if $A = B$ or $C = 0$, then only travelling waves will be formed.

(D) If the hammer is hit once,a pulse will generated and a moving pulse is a travelling wave. The pulse will more rightward, will be reflected from the wall and then move in opposite direction.

As there is no other wave, so standing waves will not form. As this is just a pulse, so particle will not perform SHM.